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A dynamic evolution model of disequilibrium network traffic flow with quantity regulation of congestion

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Abstract: Supposing that the travel cost on the paths and the congestion degree on the key links were considered by the urban travelers, a price-congestion mixed dynamic evolution model was established based on analyzing the equilibrium flow model. The model was based on the economics theory of non-Walrasian equilibrium method and by simulating the traveler's route choice behavior following the economical concept of market exploration process, the equivalency of model stability and equalization was verified. The evolution model was simulated by using a simple test network and a medium size network, the evolution process of disequilibrium network traffic flow and the performance of traffic network under the disequilibrium situation were described. Analysis result indicates that the evolution model of time price regulation accords with the classical Wardrop's first principle; the result of quantity regulation of congestion allows the degree of congestion on the key links of each path between OD to be the same; the result of price-congestion mixed regulation allows the path flow to be adjusted between the paths of lower cost and the ones of less congestion, the undulation of dynamic evolution of which is greater than that of the single regulation. In the test network, because the model only considers the choice behavior of congestion degree upon path, the congestion degree of whole traffic network is more uniform, and compared with the single price regulation model, the overall uniformity coefficient improves by 62%. However, the mean link saturation improves from 0.60 to 0.64, which indicates that the traffic network becomes congested overall. By considering the joint regulation of these two factors, the saturation of most congested link decreases from 0.936 to 0.787. The overall uniformity coefficient improves by 46%. The mean saturation of links, path travel time and congestion decrease. The test result of the medium size network also shows that such mixed equilibrium model can describe the dynamic evolution process of traffic flow on traffic network flexibly and objectively, and achieve steady state flow of traffic network system, which can

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explain the traffic travel behavior better. 8 tabs, 9 figs, 32 refs.

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一种拥挤数量调节的非均衡网络交通流动态演化模型

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摘　要：基于经济学非瓦尔拉斯均衡理论，采用经济学中市场摸索过程模拟出行者路径选择行为；假设城市出行者在路径决策过程中，考虑路径出行时间和关键路径拥挤程度的共同影响，以价格拥挤混合均衡交通流模式为基础，建立了一种价格-拥挤混合调节的非均衡网络交通流动态演化模型，并验证了模型稳定状态与均衡的等价性；基于简单的测试网络和中型路网，对演化模型进行了模拟，描述了非均衡网络交通流的演化过程与非均衡状态下交通网络的整体表现。研究结果表明：时间价格调节模型的演化结果符合经典的 Wardrop **第一原理，拥挤数量调节的结果使得** OD **间各路径上关键路段的拥挤程度一致，价格-拥挤混合调节的结果会使路径流在走行费用较小和拥挤程度较低的路径上相互进行调整，其动态演化过程波动性要大于单一调节的情况；在测试路网中，考虑采用拥挤程度对路径进行选择的行为，使得整个路网拥挤均匀程度整体提高** 62%**，但路段饱和度均值却从** 0.60 **增大到了** 0.64**，表明路网整体上变得拥挤；若考虑两者的共同调节，最拥堵路段饱和度从** 0.936 **下降到** 0.787**，均匀程度整体提高** 46%**，且路段饱和度均值降低，路径行程时间变小，拥堵得到改善；中型路网的测试结果也表明这种混合均衡模式能灵活、客观地描述路网交通流动态演化过程，获得较为合理的路网系统的稳态流量。**

关键词：交通流理论；演化模型；非瓦尔拉斯均衡；摸索过程；拥挤数量调节

0 Introduction

Traffic system is a complex open huge system. The emergence of a large number of travelers forms the distribution of traffic network flow under specific environment. Therefore, the traffic flow assignment is always one of the core problems in the field of traffic planning and management. In theory, the traffic flow of traffic network distribution will eventually reach certain equilibrium after a period of evolution. However, the traffic flow may be frequently led to a state of disequilibrium by the disturbance on traffic network caused by the sudden traffic accidents/incidents, the expected reconstruction of road, the construction of urban rail transport and the operation of road maintenance and construction area, etc. By mastering the evolution process that how traffic flow changes from the initial disequilibrium flow pattern to the final equilibrium flow pattern, a method can be provided to evaluate the overall performance of transportation network under disequilibrium condition. The basis for reliable traffic network design and traffic network control under expected or unexpected network disturbance can also be provided. In recent years, more and more scholars have been paying close attention to and studying this problem[1]. Most analysis paradigms proposed by the researchers are based on system equilibrium, which means that the traffic flow evolution rule is basically used for exploring a dynamic evolution rule of ensuring the traffic system convergence to Wardrop user optimum or system optimum equilibrium[2].

The research on the evolution model of traffic flow initially started from 1979 when Smith proposed the concept of traffic flow day-to-day (DTD) dynamic evolution. Later, many DTD evolution models were developed. These models described the personnel behavior of route choice of driver as a whole and corresponding change of network traffic flow[3]. At present, the evolution models of traffic flow mainly use the methods of analysis modeling and computer simulation[4]. The methods include simplex gravity flow dynamics[5], proportional-switch adjustment process[6-9], network tatonnement process[10], projected dynamic systems[11-12] and evolutionary game theory[13-14]. Considering social economic features and weather related variables, extensive and profound research has been carried out for DTD dynamics from emulation point of view. The above models respectively take different times to tend to be stable while simulating the traffic flow evolution process with different stabilities, but they have a common feature of travelers' rational behavior of route choice, i. e., rational behavior adjustment processes (RBAP). Specifically, in the traffic network, travelers' daily route choice behavior is a dynamic learning and game process with numerous participants. The travelers constantly get the information about traffic network to update the path travel utility cognition and adjust their own travel paths according to utility maximization principle. If all travelers' route choice behaviors are rational and ideal, it will result in the simulation process convergence. The whole process will macroscopically show the dynamic evolution of the network traffic flow from disequilibrium to equilibrium flow. Yang et al. paid attention to such ration behavior in DTD evolution model of traffic flow[15-16] and indicated that most DTD models were the dynamic evolution process ensuring the deterministic user equilibrium (DUE). In this way, when reaching the equilibrium, no vehicles in the traffic network could reduce travel cost by changing path. Guo et al. also said that limitedly rational user equilibrium can be obtained based on the limitedly rational route choice DTD model[17].

The current system equilibrium evolution model of traffic flow based on rational travel behavior can describe the evolution behavior of dynamic traffic flow under deterministic Wardrop user equilibrium very well. However, in the face of many unfavorable factors, people's judgment and decision-making behaviors do not entirely follow the Wardrop hypothesis. Furthermore, when a large number of travelers in the network are making decision at the same time, the level of carrying out rational calculation under the need of reaching the time equality and equilibrium of all paths is beyond human capacity[16]. In recent years, the heterogeneity of decision-making behavior has also caused the attention. Through experiments and studies, Brown et al. found that even in the face of the same scene, when the decision makers' expectations about the results changed, they would choose different decision schemes[18]. Tan et al. considered the travelers DTD path adjustment process and its difference and proposed a method of dynamic evolution for congestion charge strategy[19]. Xiao et al. offered a new look at the network flow dynamics, they showed that the flow dynamics is analogous to a damped oscillatory system, in terms of the aggregate effects of human behaviors[20]. Specifically, we look the day-to-day evolution of network flows that arises from travelers' route choices and their learning behavior on perceived travel costs.

Han and Du developed an ATIS-based dynamic model and studied network stability under ATIS conditions[21]. It is indicated that stability of the traffic network will be affected by both logit parameters and traffic demands of the dynamic models. Toll of related selections of roads can improve stability of the traffic network. Wu et al. believed that the existing DTD traffic distribution model is supposed to have inherent limitation[22]. Current models normally assumed that the traveler can accurately master traffic network information and always tries to minimize travel cost. In fact, the traveler is not always rational. Models established based on entirely rational assumption may get the conclusion not in line with the actual situation. Therefore, this paper studies traffic flow evolution of the urban railway network based on bounded rationality (BR) of the traveler day after day. Iryo proposed a continuous deterministic evolution model of traffic flow which took the travelers' microscopic behavior of information collection into consideration[23]. So far, most of the common documents considering the travelers' heterogeneity considered the differences caused by travelers' judgment of time value[2]. Heterogeneity exists objectively and universally and is a direct driving factor of travel behavior difference. For the study on evolution models of traffic flow, only by considering the realistic representation of travelers' non-rational behavior or heterogeneity based on traditional equilibrium analysis paradigm and establishing effective and flexible evolution model, it can be possible to explain the traffic behavior better.

So far, there are few theories about the evolution of disequilibriumtraffic flow based on the economic theories. Carey firstly introduced the terms of price and quantity in the economic disequilibrium theory when he studied the optimal models of equilibrium traffic network, but the object was the equilibrium traffic flow[24-25]. Huang et al. studied the application of disequilibrium theory in urban traffic planning, in the background of travel market, the influence of traffic spillover effect on various stages of traffic planning was analyzed, the concept and model of the disequilibrium degree about total traffic supply and demand were proposed, and the disequilibrium degree(DD) was embedded in the traffic demand forecasting model, the price-quantity regulation principle of macroeconomics disequilibrium theory was introduced into the user's route choice behavior, and a more general behavior principle was proposed as a result, however, it is not a real disequilibrium quantity regulation[26-27]. In addition, Zhang and Monden estimated the parameters of the market model based on the micro disequilibrium double market model with the method of maximum likelihood estimation[28]. Considering the process of realizing user equilibrium (UE) state in traffic network , Guo and Huang proposed a dynamical evolutionary model of traffic assignment problem under the advance traveler information systems (AITS) with endogenous origin-destionation (OD) demands by introducing the concept of decisive travel cost. The model was solved by the modified Euler method and verified on two test networks[29]. Considering the traffic network system as an economic system, Huang established the urban residents travel market based on the definition of travel demand-side and supply-side. Based on the disequilibrium theory, Huang et al. established a disequilibrium travel market, and also analyzed the adjustment mechanism of the price-and-quantity on the relationship between supply and demand in the travel market by using dynamic programming methods[30].

During the analysis on urban transportation, the actors are most sensitive to time during the process of route choice. In addition, they are extremely sensitive to the congestion degree of key link. Based on this, this paper applies the price-quantity signal regulation theory and method from the economics theory of non-Walrasian equilibrium method that the actors could not only get price signal in the market but above all could also get quantity signal, and the actors are bound by price and quantity in the market. Considering the traffic congestion regulation, this paper analyzes the influences of path travel time (price) and path congestion degree (quantity) on the distribution of traffic flow on network during the travelers' route choice process. In this paper, the independent evolution models of time price regulation and quantity regulation of congestion are first established respectively against the influences of single signals of time and congestion. Then, the mixed evolution model is established by comprehensively considering the joint regulation of price and quantity. Finally, evolution results are compared and analyzed based on these three regulations.

1 Analysis of traffic route choice

Route choice is the key for travel decision making and determining the change of dynamic traffic flow on disequilibrium network from disequilibrium to equilibrium. For urban traffic travel, travelers are most sensitive to time for route choice. Besides, for cities with serious traffic congestion problem travelers are also quite sensitive to traffic congestion degree of key links. In peak hour, many people are not willing to take the most congested path for the reason of safety and discomfort caused by driving on congested road and others, but prefer to spend more time for a less congested path.

It is supposed that the traveler considers not only travel time of the path but also the congestion degree of key links of the path in route choice decision-making process. For the purpose of this paper, key links refer to bottleneck links affected by road conditions and traffic conditions and are representative links mostly congested along the path. Low congestion degree of the routine does not mean the shortest travel time of the path. In case of low congestion degree of key links, it means that the most unsafe and most uncomfortable links can be avoided. Thus, it can be rationally explained why travelers are extremely sensitive for the most congested path for route choice.

2 Single regulation model

2.1 Evolution model based on price regulation of time

In economics, to illustrate a continuous dynamic evolution problem, it is supposed based on the network tatonnement process that commodity prices adjust in response to excess commodity demand and that the transaction price is determined based on the average transportation cost and commodity product at the market of the original place. The excessive industrial ability will be measured based on the existing market price and the difference between the existing productivity and the market demand[31]. In this section, the market exploration process in economics is adopted simulate the traveler's route choice process. The demand side in the market is the traffic demand between each OD pair. The supply side is the supply capacity of traffic network connecting OD pairs. The travel information system acts as the "auctioneer" for bidding. The supply and demand sides form the market expectation of next moment is dependent on the market information.

To give a mathematical statement of this model we express the excess transportation demand between OD pair (i, j) at time t as

$$E_{1ij}(\mu(t),h(t)) = T_{ij}(\mu(t)) - \sum_{p\in P_{ij}} h_p(t) \quad (1)$$

$$\mu(t) = (\mu_{ij}(t): i \in N_{\mathrm{O}}, j \in N_{\mathrm{D}})$$

$$h(t) = (h_p(t): p \in P)$$

$$T(\mu(t)) = (T_{ij}(\mu(t)): i \in N_{\mathrm{O}}, j \in N_{\mathrm{D}})$$

where $\mu(t)$ is minimum average travel time set at time t; $\mu_{ij}(t)$ is minimum average travel time between (i, j) at time t; $h(t)$ is route flow set at time t; $h_p(\mathrm{t})$ is flow of route p at time t; $T(\mu(t))$ is traffic demand set; $T_{ij}(\mu(t))$ is traffic demand between OD pair (i, j) at time t; N_{O}, N_{D} are sets of nodes in network; P is set of paths in network; P_{ij} is set of paths between OD pair (i, j).

The excess travel cost of the path $p \in P_{ij}$ between OD pair (i, j) at time t is defined as follow

$$E_{2p}(\mu_{ij}(t),h(t)) = c_p(h(t)) - \mu_{ij}(t) \quad (2)$$

where $c_p(h(t))$ represents the cost of path p at time t.

Assume $\mu_{ij}(t)$ is continuously differentiable as a function of time t, then

$$\frac{\mathrm{d}\mu_{ij}(t)}{\mathrm{d}t} = \lim_{\Delta t\to 0} \frac{\mu_{ij}(t+\Delta t) - \mu_{ij}(t)}{\Delta t} \quad (3)$$

The approximate value is represented as

$$\frac{\mathrm{d}\mu_{ij}(t)}{\mathrm{d}t} \approx \kappa_{ij}(\mu_{ij}(t+\Delta t) - \mu_{ij}(t)),\ \kappa_{ij} \in R \quad (4)$$

where κ_{ij} is approximate differential coefficient of $\mu_{ij}(t)$ for shortest path between OD pair (i, j); R represents the positive real number.

According to economic cobweb model, the following equation is assumed to hold

$$\mu_{ij}(t+\Delta t)=\mu_{ij}(t)+\alpha E_{1ij}(\mu(t),h(t)),\ \alpha\in R \quad (5)$$

where α represents the influence coefficient of excess traffic demand between OD pair (i, j) at the moment t on the time $\mu_{ij}(t)$ for the shortest path at time $t+\Delta t$.

Because the path time could not be negative at time $t+\Delta t$, the above equation is modified as follow

$$\mu_{ij}(t+\Delta t)=W[\mu_{ij}(t)+\alpha E_{1ij}(\mu(t),h(t))] \quad (6)$$

where $W(z)=\max(0,z)$ which prevents having negative value for z.

Substituting Eq. (6) into Eq. (4), we obtain the dynamic equation of path time as follow

$$\frac{d\mu_{ij}(t)}{dt}=\kappa_{ij}\left\{W[\mu_{ij}(t)+\alpha E_{1ij}(\mu(t),h(t))]-\mu_{ij}(t)\right\} \quad (7)$$

The same procedure may be adapted to obtain the dynamic equation of path flow as follow

$$\frac{dh_p(t)}{dt}\approx\eta_p(h_p(t+\Delta t)-h_p(t)),\ \eta_p\in R \quad (8)$$

where η_p represents the approximate differential coefficient of path flow $h_p(t)$ between OD pair (i, j).

The path flow at time $t+\Delta t$ is

$$h_p(t+\Delta t)=W[h_p(t)-\beta E_{2p}(\mu_{ij}(t),h(t))],\ \beta\in R \quad (9)$$

where β represents the influence coefficient of excess time of the path at time t on the path flow at time $t+\Delta t$.

The reason for taking a minus before parameter β is that when the travel time is higher than minimum average time, the travelers' desire to travel will reduce. Accordingly, the flow will reduce, which means that the flow-time function is negative correlation function. Substituting Eq. (9) into Eq. (8), it can be obtained

$$\frac{dh_p(t)}{dt}=\eta_p\left\{W[h_p(t)-\beta E_{2p}(\mu_{ij}(t),h(t))]-h_p(t)\right\},\ \eta_p\in R \quad (10)$$

Integrating Eqs. (8) and (10), and given the initial conditions of $\mu(t=0)=\mu^0$ and $h(t=0)=h^0$, we can obtain the price (travel time) regulation model as ($t\in[0,T]$)

$$\begin{cases}\dfrac{d\mu(t)}{dt}=\kappa\left\{W[\mu(t)+\alpha E_1(\mu(t),h(t))]-\mu(t)\right\}\\ \dfrac{dh(t)}{dt}=\eta\left\{W[h(t)-\beta E_2(\mu(t),h(t))]-h(t)\right\}\\ \mu(0)=\mu^0\\ h(0)=h^0\end{cases} \quad (11)$$

2.2 Evolution model based on quantity regulation of congestion

The congestion degree of the path $p\in P_{ij}$ between OD pair (i, j) at time t is

$$V_p(h(t))=\frac{h_p(t)}{K_p(t)} \quad (12)$$

Assume taking $K_p(t)$ as the matched traffic capacity of key link of path. The sum of matched traffic capacities of each path of link a equals to the traffic capacity of such link, where in the calculated value of matched traffic capacity of path p corresponding to each link is

$$K_p(t)=\frac{h_p(t)K_{ap}}{f_a(t)} \quad (13)$$

where $f_a(t)$ represents the link flow at time t; $K_p(t)$ is path p traffic capacity at time t.

Taking K_{ap} as the representative traffic capacity of path, equals to the smallest link traffic capacity in the path, or the defined traffic capacity by travelers

$$K_{ap}=\zeta_{ap}K_a \quad (14)$$

where traffic capacity of link a is represented as K_a, and relevance vector of the paths and the key links is represented as ζ_{ap}.

The least congested path among the paths $p\in P_{ij}$ between OD pair (i, j) is

$$\nu_{ij}(t)=\min_{p\in P_{ij}}[V_p(h(t))] \quad (15)$$

The excess congestion of each path $p\in P_{ij}$ between OD pair (i, j) is defined as

$$E_{3p}(h(t))=V_p(h(t))-\nu_{ij}(t) \quad (16)$$

The same as the reasoning of theevolution model of time price regulation, given the initial conditions of $\nu(t=0)=\nu^0$ and $h(t=0)=h^0$, we can obtain the congestion (quantity) regulation model as ($t\in[0,T]$)

$$\begin{cases}\dfrac{d\nu(t)}{dt}=\omega\left\{W[\nu(t)+\vartheta E_1(\mu(t),h(t))]-\nu(t)\right\}\\ \dfrac{dh(t)}{dt}=\eta\left\{W[h(t)-\varphi E_3(h(t))]-h(t)\right\}\\ \nu(0)=\mu^0\\ h(0)=h^0\end{cases} \quad (17)$$

where ω is approximate differential coefficient of the maximum surplus capacity ν_{ij} for the path between OD pair (i, j); ϑ is influence coefficient of excess demand of the path at time t on the maximum surplus congestion ν_{ij} at time $t+\Delta t$; φ is influence coefficient of surplus congestion of the path at time t on the path flow at time $t+\Delta t$; η is approximate differential coefficient.

3 Evolution model with mixed regulation

3.1 Price-congestion mixed equilibrium flow model

Supposing that the traveler may choose the path comprehensively based on the price (travel time) and quantity (congestion degree). The comprehensive cost of the path shall be the weighted array of the travel time cost c_p and the congestion perception cost c'_p

$$\begin{cases} \gamma = \lambda_1 c_p + \lambda_2 c'_p \\ c'_p = \sum\limits_i \theta_i c'_{pi} \\ \lambda_1 + \lambda_2 = 1 \\ 0 \leqslant \lambda_1 \leqslant 1 \\ 0 \leqslant \lambda_2 \leqslant 1 \end{cases} \tag{18}$$

where λ_1, λ_2 are weights under the joint regulation of price; c'_{pi} is congestion perception cost of the key link of i; θ_i is weight of the key link of i.

Price-congestion mixed user equilibrium (PGUE) is the lowest comprehensive cost between OD among all paths which is smaller than or equal to the comprehensive cost of any other unused path.

When $\lambda_1=1$ and $\lambda_2=0$, the travel will choose the path only based on the travel cost. PGUE will be the simple price regulation user equilibrium (PUE), the lowest cost among all used paths between OD, lower than or equal to travel cost of any other paths unused. Of course, PUE is the well known Wardrop user equilibrium.

When $\lambda_1=0$ and $\lambda_2=1$, the traveler will choose the path only based on the congestion degree. PGUE will be the simple congestion quantity regulation user equilibrium (OUE), the lowest congestion degree among all used paths between OD.

Different equilibrium flow models can be obtained through different route choice behavior assumption. When the price-congestion quantity regulation principle is applied for individual travel route choice, it means that the individual will select a path based on comprehensive consideration of travel time and path comfort degree. When it is applied for a group, it comes that a part of travelers choose the shortest path for travel while a part of travelers choose the path with the lowest congestion degree for travel. The values of λ_1, λ_2 shall be determined based on individual or group characteristics.

3.2 Evolution model with price-congestion mixed regulation

Given the initial conditions of $\mu(t=0)=\mu^0$, $\nu(t=0)=\nu^0$ and $h(t=0)=h^0$, the weighed price-congestion mixed regulation model can be obtained as ($t\in[0, T]$)

$$\begin{cases} \dfrac{\mathrm{d}\mu(t)}{\mathrm{d}t}=\kappa\Big\{W[\mu(t)+\alpha E_1(\mu(t),h(t))]-\mu(t)\Big\} \\ \dfrac{\mathrm{d}\nu(t)}{\mathrm{d}t}=\omega\Big\{W[\nu(t)+\vartheta E_1(\mu(t),h(t))]-\nu(t)\Big\} \\ \dfrac{\mathrm{d}h(t)}{\mathrm{d}t}=\eta\Big\{\lambda_1 W[h(t)-\beta E_2(\mu(t),h(t))]+ \\ \qquad \lambda_2 W[h(t)-\varphi E_3(h(t))]-h(t)\Big\} \\ \mu(0)=\mu^0 \\ \nu(0)=\nu^0 \\ h(0)=h^0 \end{cases} \tag{19}$$

3.3 Equivalency of model stability and equalization

When the evolution model Eq. (19) is stable, the travel time, path congestion degree and path flow dynamic evolution state shall be

$$\frac{\mathrm{d}\mu(t)}{\mathrm{d}t}=0\Rightarrow\mu(t)=W[\mu(t)+\alpha E_1(\mu(t),h(t))] \tag{20}$$

$$\frac{\mathrm{d}\nu(t)}{\mathrm{d}t}=0\Rightarrow\nu(t)=W[\nu(t)-\vartheta E_1(\mu(t),h(t))] \tag{21}$$

$$\frac{\mathrm{d}h(t)}{\mathrm{d}t}=0\Rightarrow h(t)=W[h(t)-(\beta\lambda_1 E_2(\mu(t),h(t))+ \\ \varphi\lambda_2 E_3(h(t)))] \tag{22}$$

The three fixed points may be expressed by the variational inequality below

$$\sum_{ij}\Big(\sum_{p\in P_{ij}} h_p^* - T_{ij}(\mu^*)\Big)(\mu_{ij}-\mu_{ij}^*)+ \\ \sum_{ij}\Big(T_{ij}(\mu^*)-\sum_{p\in P_{ij}} h_p^*\Big)(\nu_{ij}-\nu_{ij}^*)+ \\ \sum_{ij}\sum_{p\in P_{ij}}\Big\{\beta\lambda_1[c_p(h^*)-\mu_{ij}^*]+ \\ \varphi\lambda_2[\nu_{ij}^*-V_p(h^*)]\Big\}(h_p-h_p^*)\geqslant 0 \tag{23}$$

KKT conditions of the variational inequality Eq. (23) are as follows

$$\beta\lambda_1[c_p(h^*)-\mu_{ij}^*]+\varphi\lambda_2[\nu_{ij}^*-V_p(h^*)]-\xi_p=0,\ \xi_p\geqslant 0,\ \xi_p h_p^*=0 \tag{24}$$

$$\sum_{p\in P_{ij}} h_p^*-T_{ij}(\mu^*)-\varepsilon_{ij}=0,\ \varepsilon_{ij}\geqslant 0,\ \varepsilon_{ij}\mu_{ij}^*=0 \tag{25}$$

$$T_{ij}(\mu^*)-\sum_{p\in P_{ij}} h_p^*-\psi_{ij}=0,\ \psi_{ij}\geqslant 0,\ \psi_{ij}v_{ij}^*=0 \tag{26}$$

where ξ_p, ε_{ij} and ψ_{ij} are nonnegative restricted dual variable vectors; the parameters with " $*$ " are vector solutions for variational inequality.

Meanings of KKT conditions are discussed below in different conditions.

(a) If $\lambda_1=1$, $\lambda_2=0$, Eq. (24) will change into $\beta[c_p(h^*)-\mu_{ij}^*]-\xi_p=0$. When equilibrium flow $h_p^*>0$, $\xi_p=0$, it can be concluded that $c_p(h^*)=\mu_{ij}^*$; when equilibrium flow $h_p^*=0$, $\xi_p>0$, it can be concluded that $c_p(h^*)>\mu_{ij}^*$. Eq. (24) is equivalent to Wardrop user equivalence. Since $\mu_{ij}^*=\min\limits_{p\in P_{ij}} c_p(h^*)>0$, $\varepsilon_{ij}=0$, Eq. (25) is the flow conservative restriction.

(b) If $\lambda_1=0$, $\lambda_2=1$, Eq. (24) will turn to $\varphi[v_{ij}^*-V_p(h^*)]-\xi_p=0$. When equilibrium flow $h_p^*>0$, it can be concluded that $v_{ij}^*=V_p(h^*)$; when equilibrium flow $h_p^*=0$, it can be concluded that $v_{ij}^*>V_p(h^*)$. Eq. (24) is equivalent to single congestion constraint. Since $K_p\geqslant h_p$, $v_{ij}=\max[V_p(h^*)]>0$ and $\psi_{ij}=0$. Then, Eq. (26) is the flow conservative restriction.

(c) If λ_1 and λ_2 is in the scope of (0, 1), Eq. (24) is equivalent to price-congestion fixed restriction. Eqs. (25) and (26) shall be flow conservative restriction.

It is known from the above analysis, when the path's comprehensive cost is positive and continuous and when the demand function is continuous, the stability of the price-congestion mixed regulation evolution model and the price-congestion mixed regulation user equalization flow mode is equivalent.

4 Example analysis

To illustrate some of the ideas discussed above, we begin with the simple network illustrated in Fig. 1[10]. There are three paths between the single OD pair (1, 4) in Fig. 1

$$p_1=\{a_1,a_4\}$$

$$p_2=\{a_2,a_5\}$$

$$p_3=\{a_1,a_3,a_5\}$$

The link cost function is

$$c_a(f_a)=A_a+B_a(f_a(t)/K_a)^4$$

where p_1-p_3 are paths; a_1-a_5 are links; A_a, B_a are coefficient variations.

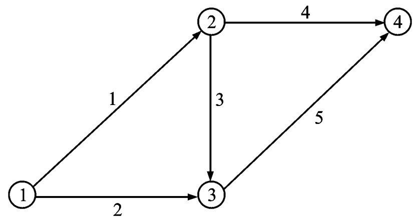

Fig. 1 Test traffic network[10]

The link-path incidence matrix $\boldsymbol{\Delta}$ is shown as

$$\boldsymbol{\Delta}=\begin{bmatrix}1&0&1\\0&1&0\\0&0&1\\1&0&0\\0&1&1\end{bmatrix}$$

The cost function parameters are shown in Tab. 1, and the regulation model parameters are shown in Tab. 2.

Tab. 1 Cost function parameters

Link	A_a	B_a	K_a
1	4	0.60	40
2	6	0.90	50
3	2	0.30	40
4	5	0.75	30
5	3	0.45	50

Tab. 2 Evolution model parameters

Parameter	α	β	κ	η	ϑ	φ
Value	0.5	2.0	1.0	1.0	0.5	2.0

Given the fixed demand of $T=60$ and the initial path flow of $h(0)=\{0,0,0\}$, the initial cost $\mu(0)$ is determined by $h(0)$, the evolution is carried out for $t\in[0,\ 200]$, the step is taken as 0.05, and it means that the evolutionary time domain is discreted into 4 000 periods to calculate. The value of the time domain is simulated, and the actual evolution time can be selected according to the time of loading traffic flow once in the network. The values of above three models are respectively calculated as follows.

4.1 Single price regulation of time

The path flow evolution process under single price regulation is shown in Figs. 2 and 3, which respectively represents the dynamic evolution processes of path flows h_1, h_2 and h_3.

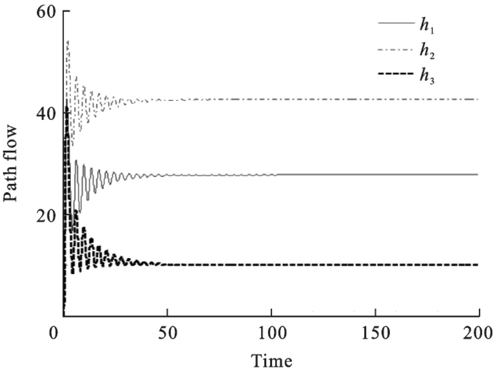

Fig. 2 Dynamic adjustment process of price regulation

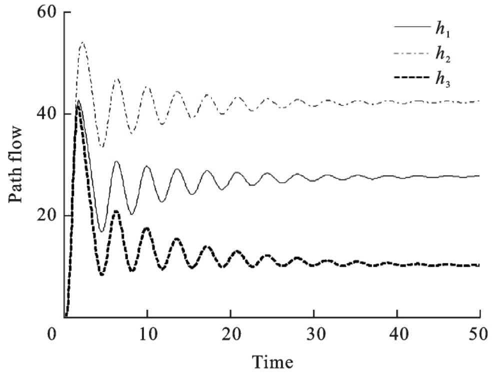

Fig. 3 Local dynamic adjustment process of price regulation

Tab. 3 shows that under the single price regulation, the travel costs of different paths reach unanimity when coming to a steady state.

Tab. 3 Fixed demand equilibrium network path flow, cost and maximum congestion degree

Path	Link	Path flow	Path cost	Path congestion degree
h_1	1, 4	28.082	9.414	0.936
h_2	2, 5	28.478	9.414	0.638
h_3	1, 3, 5	3.440	9.414	0.788

4.2 Single quantity regulation of congestion

The path flow evolution process under single congestion regulation is shown in Figs. 4 and 5 with coincident lines, which indicates the consistent dynamic evolution rule of path flows h_1 and h_2.

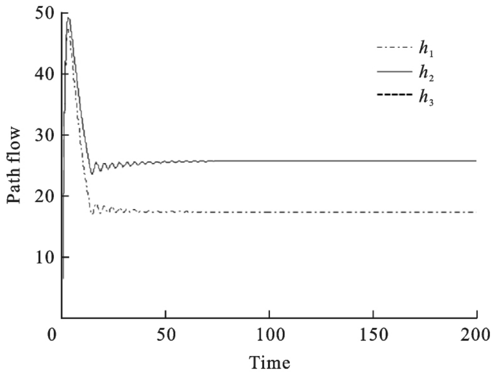

Fig. 4 Dynamic adjustment process of congestion regulation

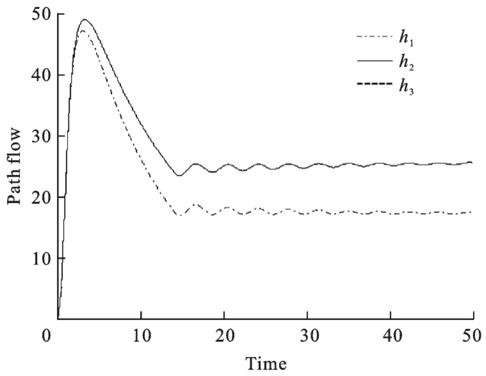

Fig. 5 Local dynamic adjustment process of congestion regulation

Tab. 4 shows that under the single congestion regulation, the congestion degrees (saturation levels) of key links of different paths are consistent when coming to a steady state. Because the key links of paths 2 and 3 are consistent under this traffic network, the path flow distributions are consistent, which results in the coincidence of their dynamic evolution process figures.

Tab. 4 Fixed demand equilibrium network path flow and maximum congestion degree

Path	Link	Path flow	Path congestion degree
h_1	1, 4	17.143	0.857
h_2	2, 5	25.714	0.857
h_3	1, 3, 5	17.143	0.857

4.3 Price-congestion mixed regulation

Under the price-congestion mixed regulation, it is supposed that the weight λ_1 occupied by the influence of price factor is taken as 95%, and the weight λ_2 occupied by congestion factor is taken as 5%. The path flow evolution process is shown in Figs. 6 and 7.

Tab. 5 reflects the dynamic evolution process of traffic flow under price-congestion mixed regulation. Its undulation is greater than that of the single regulation, and the convergence speed is the slowest.

4.4 Comparative analysis

The model calculation result shows that three

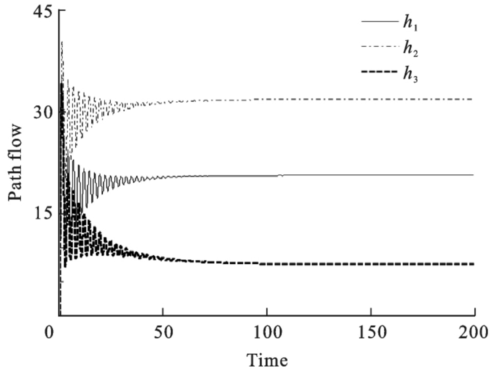

Fig. 6 Dynamic adjustment process of mixed regulation

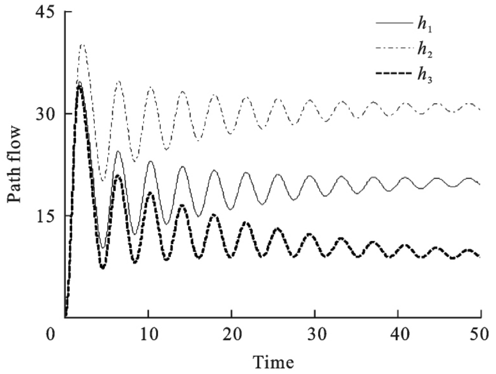

Fig. 7 Local dynamic adjustment process of mixed regulation

regulation models can all simulate the process of path flow changing along with time relatively ideally, and all path flows will maintain a stable state after a certain time of undulation. The difference between the path flows in Tabs. 3 and 4 shows that considering the influence of congestion degree on route choice will result in more balanced flows of paths. Through further calculating the link saturation (Tab. 6), it is found that compared with single price regulation model, the congestion regulation relieves the most congested link obviously and allows the congestion degrees of links to be more balanced, with the overall uniformity coefficient improving by 62%. However, this model only considers the choice behavior of congestion degree upon path, which allows the congestion degree of the whole traffic network to be more uniform while improving the mean link saturation from 0.60 to 0.64. It indicates that the traffic network becomes congested overall. In case of considering the joint regulation of these two factors, a relatively satisfactory result can be achieved. The saturation of most congested link decreases from 0.936 to 0.787. The overall uniformity coefficient improves by 46%. The mean saturation of links and the path travel time decrease. Consequently, the overall congestion condition of the traffic network becomes more uniform, and the congestion reduces.

Tab. 5 Fixed demand equilibrium network path flow, cost and maximum congestion degree

Path	Link	Path flow	Path cost	Path congestion degree
h_1	1, 4	20.654	9.316	0.704
h_2	2, 5	31.832	9.320	0.787
h_3	1, 3, 5	7.513	9.320	0.787

Tab. 6 Calculated values of saturation of road link under different regulation mechanisms

Link	1	2	3	4	5	Mean	Range	Variance	Standard deviation
Price regulation	0.788 1	0.569 6	0.086 0	0.936 1	0.638 4	0.603 6	0.850 1	0.103 7	0.322 0
Congestion regulation	0.857 2	0.514 3	0.428 6	0.571 4	0.857 1	0.645 7	0.428 6	0.039 8	0.199 6
Mixed regulation	0.704 2	0.636 6	0.187 8	0.688 5	0.786 9	0.600 8	0.599 1	0.056 2	0.237 1

4.5 Medium size network test

In order to verify the operation effect of the model in a more complex network, as shown in Fig. 8, a medium test traffic network is selected, including 13 network nodes, 19 links and 4 OD pairs.

In the test, considering the complexity of traffic network, the weight of the influence of congestion factor increases to 10%. Given the fixed demand of $(T_{12}, T_{13}, T_{42}, T_{43}) = (35, 25, 20, 25)$ and the parameters of each link cost function in Tab. 7[32].

After a period of simulation, all path flows and

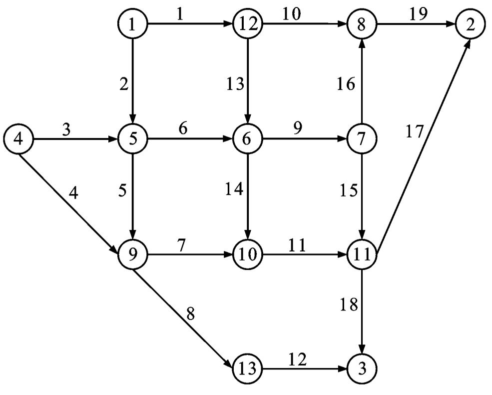

Fig. 8 Medium size test traffic network

Tab. 7 Parameters of each link cost function

Link	1	2	3	4	5	6	7
A_a	8.00	5.00	2.00	6.00	4.00	5.00	3.50
B_a	0.060	0.090	0.030	0.075	0.045	0.060	0.030
K_a	30	30	30	30	30	40	30
Link	8	9	10	11	12	13	14
A_a	2.00	5.00	8.02	5.40	8.00	2.00	2.50
B_a	0.030	0.075	0.055	0.060	0.050	0.030	0.085
K_a	50	30	30	30	30	50	40
Link	15	16	17	18	19		
A_a	3.00	1.02	4.00	1.00	6.00		
B_a	0.045	0.080	0.090	0.040	0.075		
K_a	30	30	40	50	30		

cost are maintained at a stable state. Tab. 8 gives the path flows，path costs and congestion degrees of different OD pairs. The process of dynamic evolution of partial path flows is shown in Fig. 9.

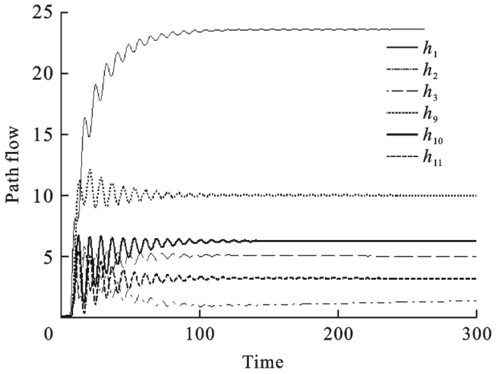

Fig. 9 Dynamic adjustment process of partial path flows

Tab. 8 gives the results of dynamic evolution model with mixed regulation in the complex network，which shows that the mixed regulation model is also applicable to the actual complex network. During practical transportation process，people would not necessarily all choose the shortest path according to time，even if they have fully mastered all kinds of traffic informations. In particular，in rush hours，many people would rather spend more time to go by a roundabout path to drive on the less congested road than choosing the most congested link. Therefore，the price-congestion mixed regulation model can take this intention of travelers into consideration more flexibly and can explain the transportation behavior more reasonably.

Tab. 8 Fixed demand equilibrium network path flow，cost and maximum congestion degree

OD pair	Path	Link	Path flow	Path cost	Path congestion degree
(1，2)	h_1	1，10，19	23.510 4	23.961 9	0.827 4
	h_2	2，6，9，16，19	1.270 2	23.961 9	0.827 4
	h_3	2，6，9，15，17	4.913 7	23.955 9	0.881 4
	h_4	2，6，14，11，17	1.867 5	23.948 5	0.947 6
	h_5	2，5，7，11，17	1.321 6	23.948 5	0.947 6
	h_6	1，13，9，16，19	0.023 1	23.961 9	0.881 4
	h_7	1，13，9，15，17	2.054 3	23.955 9	0.881 4
	h_8	1，13，14，11，17	0.042 3	23.948 5	0.947 6
(1，3)	h_9	2，5，8，12	9.904 9	20.783 3	0.749 7
	h_{10}	2，6，9，15，18	6.131 7	20.761 3	0.881 4
	h_{11}	2，6，14，11，18	3.083 5	20.768 7	0.947 6
	h_{12}	2，5，7，11，18	2.239 9	20.761 3	0.947 6
	h_{13}	1，13，9，15，18	3.272 5	20.768 7	0.881 4
	h_{14}	1，13，14，11，18	0.367 5	20.761 3	0.947 6
(4，2)	h_{15}	4，7，11，17	7.855 5	20.217 8	0.947 6
	h_{16}	3，6，9，16，19	0.019 3	20.231 2	0.881 4
	h_{17}	3，6，9，15，17	5.859 6	20.225 2	0.881 4
	h_{18}	3，6，14，11，17	3.647 7	20.217 8	0.947 6
	h_{19}	3，5，7，11，17	2.617 9	20.217 8	0.947 6
(4，3)	h_{20}	4，8，12	11.402 9	17.030 6	0.749 8
	h_{21}	4，7，11，18	3.236 8	17.052 6	0.947 6
	h_{22}	3，5，8，12	5.312 8	17.052 6	0.749 7
	h_{23}	3，6，9，15，18	2.896 7	17.038 0	0.881 4
	h_{24}	3，6，14，11，18	1.057 6	17.030 6	0.947 6
	h_{25}	3，5，7，11，18	1.093 3	17.030 6	0.947 6

5 Conclusions

All the three dynamic models based on economic equilibrium theory indicate that after a period of undulation, the path flow of traffic network will maintain a steady state without any change, and the network traffic system is in equilibrium. The evolution result of time price regulation model accords with the classical Wardrop's first principle for transportation planning. The result of mixed price-congestion regulation is the mixed equilibrium of price regulation equilibrium model and congestion regulation equilibrium model, which allows the path flow direction to be mutually adjusted with the paths with less travel cost and low congestion degree.

Compared with single price regulation model, the model considering the choice behavior of congestion degree upon path allows the congestion degree of the whole traffic network to be more uniform. This provides a new idea about making strategies for network traffic flow control and guidance and for traffic construction plan.

The study in this paper is an innovative attempt to provide a new analysis paradigm and research conclusions available for referenceof research on traffic flow dynamic evolution. However, there is still some work to be further carried out. For example, the initial value of the evolution model is arbitrarily given and is not calibrated based on the actual traffic network state, the evolution cannot really reflect actual traffic flow change. Related empirical study in the future can be beneficial for parameter calibration. Besides, modeling based on non-Walrasian equilibrium theory, the shift from nominal demand to effective demand, the way of considering demand change and the applications in the aspects of dynamic traffic control, etc. are all the work directions that we will research in the future.

References:

[1] BOYCE D. Beckmann's transportation network equilibrium model: Its history and relationship to the Kuhn-Tucker conditions[J]. Economics of Transportation, 2013, 2(1): 47-52.

[2] XU Hong-li. Urban transportation network equilibrium and congestion pricing with bounded rationality[D]. Nanjing: Nanjing University, 2011.

[3] KUMAR A, PEETA S. A day-to-day dynamical model for the evolution of path flows under disequilibrium of traffic networks with fixed demand[J]. Transportation Research Part B: Methodological, 2015, 80: 235-256.

[4] HE Xiao-zheng, PEETA S. Amarginal utility day-to-day traffic evolution model based on one-step strategic thinking [J]. Transportation Research Part B: Methodological, 2016, 84: 237-255.

[5] SMITH M J. The existence and calculation of traffic equilibria[J]. Transportation Research Part B: Methodological, 1983, 17(4): 291-303.

[6] SMITH M J. The stability of a dynamic model of traffic assignment-an application of a method of Lyapunov[J]. Transportation Science, 1984, 18(3): 245-252.

[7] SMITH M J, WINSTERN M B. A continuous day-to-day traffic assignment model and the existence of a continuous dynamic user equilibrium [J]. Annals of Operations Research, 1995, 60(1): 59-79.

[8] HUANG Hai-jun, LAM W. Modeling and solving the dynamic user equilibrium route and departure time choice problem in network with queues[J]. Transportation Research Part B: Methodological, 2002, 36(3): 253-273.

[9] PEETA S, YANG T H. Stability issues for dynamic traffic assignment[J]. Automatica, 2003, 39(1): 21-34.

[10] FRIESZT L. Day-to-day dynamic network disequilibria and idealized traveler information systems[D]. Fairfax: George Mason University, 1994.

[11] ZHANG Ding, NAGURNEY A. On the local and global stability of a travel route choice adjustment process[J]. Transportation Research Part B: Methodological, 1996, 30(4): 245-262.

[12] ZHANG Ding, NAGURNEY A. On the equivalence between stationary link flow patterns and traffic network equilibria[J]. Transportation Research Part B: Methodological, 2001, 35(8): 731-748.

[13] SANDHOLM W H. Potential games with continuous player sets[J]. Journal of Economic Theory, 2001, 97(1): 81-108.

[14] YANG F. An evolutionary game theory approach to the day-to-day traffic dynamics[D]. Madison: University of Wisconsin-Madison, 2005.

[15] YANG Fan, ZHANG Ding. Day-to-day stationary link flow pattern[J]. Transportation Research Part B: Methodological, 2009, 43(1): 119-126.

[16] TARUN R, BOYLES S D. Dynamic pricing in discrete time stochastic day-to-day route choice models[J]. Transportation Research Part B: Methodological, 2016, 92: 104-118.

[17] GUO Ren-yong, YANG Hai, HUANG Hai-jun. Day-to-day flow dynamics and congestion control[J]. Transportation Science, 2016, 50(3): 982-997.

[18] BROWN D B, GIORGI E D, SIM M. Aspirational preferences

and their representation by risk measures[J]. Management Science, 2012, 58(11): 2095-2113.

[19] TAN Zhi-jia, YANG Hai, GUO Ren-yong. Dynamic congestion pricing with day-to-day flow evolution and user heterogeneity[J]. Transportation Research Part C: Emerging Technologies, 2015, 61: 87-105.

[20] XIAO Feng, YANG Hai, YE Hong-bo. Physics of day-to-day network flow dynamics[J]. Transportation Research Part B: Methodological, 2016, 86: 86-103.

[21] HAN Lan-shan, DU Li-li. On a link-based day-to-day traffic assignment model[J]. Transportation Research Part B: Methodological, 2012, 46(1): 72-84.

[22] WU Jian-jun, SUN Hui-jun, WANG D Z W, et al. Bounded-rationality based day-to-day evolution model for travel behavior analysis of urban railway network[J]. Transportation Research Part C: Emerging Technologies, 2013, 31: 73-82.

[23] IRYO T. Day-to-day dynamical model incorporating an explicit description of individuals' information collection behaviour[J]. Transportation Research Part B: Methodological, 2016, 92: 88-103.

[24] CAREY M. The dual of the traffic assignment problem with elastic demands[J]. Transportation Research Part B: Methodological, 1985, 19(3): 227-237.

[25] CAREY M. Network equilibrium: optimization formulations with both quantities and prices as variable[J]. Transportation Research Part B: Methodological, 1987, 21(1): 69-77.

[26] HUANG Zhong-xiang, HE Guo-guang, LIU Bao. Disequilbrium traffic assignment models by price-quantity regulation [J]. Journal of System Engineering, 1999, 14(2): 145-151.

[27] HUANG Zhong-xiang, HE Guo-guang, LIU Bao. A primary study on disequilibrium transportation planning[J]. Journal of Management Sciences in China, 2001, 4(1): 52-57.

[28] ZHANG Jun-yi, MONDEN H. Transport demand and supply model based on disequilibrium theory[J]. Journal of the Eastern Asia Society for Transportation Studies, 2003(5): 1262-1277.

[29] GUO Ren-yong, HUANG Hai-jun. Dynamical evolutionary model of traffic assignment under AITS[J]. Journal of Management Sciences in China, 2008, 11(2): 12-19.

[30] HUANG Zhong-xiang, CAI Jian-rong, WU Li-xuan. Disequilibrium adjustment mechanism for urban resident travel markets[J]. Journal of Chang'an University: Natural Science Edition, 2016, 36(6): 105-110.

[31] FRIESZ T L, SUO Zhong-gui, BERNSTEIN D. A dynamic disequilibrium interregional commodity flow model [J]. Transportation Research Part B: Methodological, 1998, 32(7): 467-483.

[32] NGUYEN S, DUPIU C. An efficient method for computing traffic equilibria in networks with asymmetric transportation costs[J]. Transportation Science, 1984, 18(2): 185-202.